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QCD, Supersymmetric QCD, Lattice QCD and String Theory: Synthesis on the Horizon?

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Supersymmetric gauge theories in four dimensions have taught us many important physics lessons. These can both inform and be informed by future work on the lattice. I focus on three issues: the properties of supersymmetric Yang-Mills theory and its relation to the non-supersymmetric case; the properties of gauge theories with matter and their relation to real QCD; and, briefly, the recent discovery that gauge theories and string theories are more deeply connected than ever previously realized. Specific questions for lattice gauge theorists to consider are raised in the context of the first two topics.

1. Introduction

The field of supersymmetric gauge theory is vast, fascinating, and technical, and obviously cannot be reviewed in an hour. My limited intention today is to give a qualitative and conceptual talk, with an eye toward conveying what I see as the most important messages for lattice gauge theory arising from the recent advances in this branch of mathematical physics. Consequently, this lecture will consist merely of results, largely those of other authors; it will neither contain descriptions of the evidence for these results nor any technical details, and referencing will be limited. I learned much of the physics in conversations with Seiberg and with my collaborators Intriligator and Leigh. However, the synthesis presented here has not been emphasized elsewhere.

The purpose of my giving such a lecture to an audience of lattice gauge theorists? This is perhaps conveyed best by Tom DeGrand's words: he requested that I encourage you "to change the line in [your] code that sets $N_c, N_f = 3$." I hope to convince you that a systematic exploration of

theories *other* than real-world QCD is an important and exciting direction for research, and that placing QCD in the context of a wider variety of theories may become a powerful tool for understanding it.

The topics I will cover today are the following. First, I will discuss $\mathcal{N} = 1$ super-Yang-Mills (SYM) theory and compare it with Yang-Mills (YM) theory and with QCD (by which I mean YM with matter — unless otherwise specified, fermions in the fundamental representation.) Second, I will discuss SQCD ($\mathcal{N} = 1$ SYM with fermions and scalars in the fundamental representation) and its relation to QCD. And finally, I will briefly discuss the recently discovered connection relating gauge theory to gravity and string theory. (I have also written lectures for non-experts in [1] and [2]; parts of the present talk have been abridged because of overlap with the previous articles.)

1.1. Beyond QCD

There are several reasons why physicists should seek a detailed understanding of gauge theories beyond real-world QCD. First, we can use them to gain insight into the properties of the real world. It would be helpful to know which as-

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pects of the strong interactions are special to $N_c = N_f = 3$ and which ones are generic, or at least common to many models. Second, a theory with behavior only vaguely similar to QCD may be responsible for electroweak symmetry breaking, as in technicolor and topcolor models. Examples of non-QCD-like theories are the fixed-point models discussed in [3]. Third, it is important to test numerically some of the analytic predictions of supersymmetric gauge theory, in part to close some remaining loopholes in the analytic arguments. And finally, there are possible applications to condensed matter.

Four-dimensional supersymmetric gauge theories are good toy models for non-supersymmetric QCD and its extensions. Some of these theories display confinement; of these, some break chiral symmetry but others do not [4,5]. The mechanism of confinement occasionally can be understood using a weakly-coupled “dual description” [6–8] (an alternate set of variables for describing the same physics.) However, what is more striking is that most of these theories do not confine [8]! Instead, their infrared physics is governed by other, unfamiliar phases, often described most easily using dual variables. (Here and throughout, “phase” refers to the properties of the far infrared physics at zero temperature.) The phase diagram, as a function of the gauge group, matter content, and interactions, is complex and intricate [8]. Recently, it has been found that the large- N_c physics of these theories may be tractable — and that it is superstring theory [9–12]! I will discuss all of these issues below.

2. Pure $\mathcal{N} = 1$ Super-Yang-Mills

Consider $SU(N)$ gauge theory with a vector boson A_μ and a Majorana spinor λ_α , with Lagrangian

$$\mathcal{L} = \frac{1}{2g^2} [\text{tr } F_{\mu\nu} F^{\mu\nu} + i\bar{\lambda}\not{D}\lambda] \quad (1)$$

This is $\mathcal{N} = 1$ SYM theory. It exhibits chiral symmetry breaking: a gluino condensate $\langle\lambda\lambda\rangle$ forms, leading to N equivalent vacua, differing only by the phase of the condensate, which rotate into one another under $\theta \rightarrow \theta + 2\pi$. Domain walls can

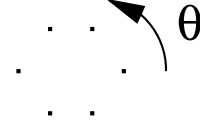


Figure 1. The vacua of SYM theory are rotated by the theta angle.

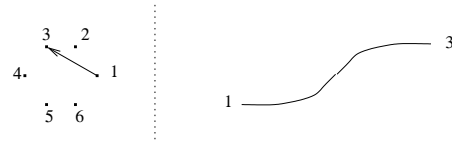


Figure 2. Domain walls can interpolate between vacua of SYM theory.

separate regions of different vacua. The theory confines and exhibits electric flux tubes.

2.1. Chiral Symmetry Breaking

Let us examine the issue of chiral symmetry breaking. The classical $U(1)$ axial symmetry $\lambda \rightarrow \lambda e^{i\sigma}$ is anomalous; since λ has $2N$ zero modes in the presence of an instanton, only a \mathbf{Z}_{2N} , under which $\lambda \rightarrow \lambda e^{i\pi n/N}$, is anomaly-free. The dynamics leads to N equivalent vacua with $\langle\lambda\lambda\rangle = \Lambda^3 e^{i(2\pi n/N)}$, which breaks the \mathbf{Z}_{2N} to a \mathbf{Z}_2 under which $\lambda \rightarrow -\lambda$. Note that an instanton in this theory comes with a factor Λ^{3N} ; the form of the gluino condensate suggests it is induced by $1/N$ fractional instantons.

If we shift the theta angle by $\theta \rightarrow \theta + \alpha$, then $\lambda \rightarrow \lambda e^{i\alpha/2N}$, and the N equivalent vacua (Fig. 1) are rotated by an angle α/N . Any given vacuum only comes back to itself under $\theta \rightarrow \theta + 2\pi N$, but the physics is invariant under $\theta \rightarrow \theta + 2\pi$. Domain walls can exist between regions in different vacua; as in Fig. 2, the condensate $\langle\lambda\lambda\rangle$ can change continuously from $\Lambda^3 e^{2\pi i n/N}$ to $\Lambda^3 e^{2\pi i n'/N}$.

It is important to note that similar properties — equivalent discrete vacua, domain walls, and possible fractional instantons — might oc-

cur (and should be searched for) in certain non-supersymmetric gauge theories with fermions.

2.2. Confinement and Flux Tubes

[I have abbreviated some of the content of this section; the interested reader can find more detail in [1].]

$\mathcal{N} = 1$ SYM is confining, with electric flux tubes which as always carry charge in the center of the gauge group; for $SU(N)$ they carry quantum numbers k in \mathbf{Z}_N . The tension T_k of these confining strings is a function of k, N, Λ ; note $T_k = T_{N-k}, T_N = 0$ by \mathbf{Z}_N symmetry. An electric source of charge k (*e.g.* one in a k -index antisymmetric tensor representation) will be confined by a k -string (a string carrying k units of flux.) The ratio $T_k/T_{k'}$ is a basic property of YM and SYM, as fundamental as the hadron spectrum and easier for theorists to estimate. Two interesting predictions for this ratio are obtained through the Hamiltonian strong coupling expansion, which gives the second Casimir of the representation

$$T_k \propto k(N-k)/N \quad (2)$$

and through weakly broken $\mathcal{N} = 2$ supersymmetric gauge theory [7]

$$T_k \propto \sin \frac{\pi k}{N} \quad (3)$$

A question of considerable interest is whether either formula well describes SYM and/or YM theory.

A more qualitative question is whether YM/SYM is a Type I or Type II dual superconductor — that is, whether $T_2 < 2T_1$ or $T_2 > 2T_1$. In the former case flux tubes attract one another, in the other they repel. On general grounds I personally expect that $2T_1 > T_2 > T_1$ for $SU(N)$, $N > 4$. The reasoning for this is the following. For $N = 2$, we have $T_2 = 0$, while for $N = 3$ we have $T_2 = T_1, T_3 = 0$ on general grounds. For large N , we would expect $T_k = kT_1 \pm \mathcal{O}(1/N)$ since a k -string is a bound (or unbound) state of k 1-strings, but string-string interactions are of order $1/N$. Any reasonable interpolating formula will satisfy $kT_1 > T_k > T_1$ for $N > 4$, $1 < k < N - 1$. Ohta and Wingate [13] have

compared T_1 and T_2 in $SU(4)$ by computing the potential energy between sources in the $\mathbf{4}$ and $\bar{\mathbf{4}}$ representation ($V_{4\bar{4}}(r) \sim T_1 r$) and the energy between sources in the $\mathbf{6}$ representation ($V_{66}(r) \sim T_2 r$.) Their preliminary results indicate indeed that $2T_1 > T_2 > T_1$.

2.3. Mechanism of Confinement

[The material of this section was carefully covered in [1], and has been heavily abridged. I have attempted here only to outline the results, and focus attention on the physics message.]

What drives confinement? This question cannot be addressed directly in SYM, because this theory does not have a weakly-coupled dual description. However, there is a trick for studying this question — within limits, as discussed further below. The trick is the following. One can add additional massive matter to SYM without leaving its universality class (note supersymmetry and holomorphy ensure this; see [5]) In particular I will study broken $\mathcal{N} = 2$ SYM [6,7] and broken $\mathcal{N} = 4$ SYM [14,15,1] gauge theories which have the same massless fields as $\mathcal{N} = 1$ SYM. These theories have a duality transformation which allows their dynamics to be studied using a weakly coupled magnetic description. In this description it will be possible to show that there are monopoles in the theory, which condense, thereby causing confinement [6]. The required \mathbf{Z}_N flux tubes will appear as string solitons. The picture which emerges will strongly support the old lore on the Dual Meissner effect.

However, a strong word of caution is in order here. In particular, although these theories are in the same universality class as $\mathcal{N} = 1$ SYM, they are not equivalent to it. While confinement and an energy gap are universal properties of all of these theories, the monopoles which lead to confinement are *not* universal. The properties of the monopoles will depend on the matter that is added to SYM. We will shortly see that this poses problems for abelian projection approaches to confinement.

I will now illustrate these points by studying the broken $\mathcal{N} = 2$ and $\mathcal{N} = 4$ SYM theories. The $\mathcal{N} = 2$ $SU(N)$ SYM theory, with strong coupling scale Λ , has a dual description as an abelian

$U(1)^{N-1}$ gauge theory [6, 16–18]; the monopoles of the $SU(N)$ description are electrically charged under the dual description. When masses $\mu \ll \Lambda$ are added so that the only massless fields are those of $\mathcal{N} = 1$ SYM, the monopoles develop expectation values. The dual description of this condensation involves nothing more than $N - 1$ copies of the Abelian Higgs model, and so gives $N - 1$ solitonic Nielsen-Olesen strings [6, 7], each carrying an integer charge. Electrically charged sources are therefore confined, and the flux tubes which confine them are the solitonic strings of the dual description. However, the confining strings are problematic [7, 19]. Although they carry an exact \mathbf{Z}_N symmetry, they also each carry an (approximate) integer charge, violated only at the scale Λ which is large compared to the string tension. This extra symmetry leads the theory to exhibit not one but $N/2$ Regge trajectories — thus the theory does not have the same properties as YM or $\mathcal{N} = 1$ SYM. As $\mu \rightarrow \Lambda$, the extra symmetry begins to disappear, but at the same time the magnetic description becomes strongly coupled, so no reliable dual description can be given in the regime where the theory behaves as $\mathcal{N} = 1$ SYM is expected to do.

Note these properties are characteristic of abelian projection approaches to confinement. If we project $SU(N) \rightarrow U(1)^{N-1}$, dynamically or otherwise, then abelian monopole condensation leads to Nielsen-Olesen strings, each with its own approximately conserved integer charge. This unavoidable charge inhibits annihilation of N identical strings (as in Fig. 3) which both leads too an overabundance of metastable hadrons and to difficulty in forming baryons [19]. This is a serious problem for abelian projection approaches to QCD.

The situation in broken $\mathcal{N} = 4$ SYM [1] is much more satisfactory. $\mathcal{N} = 4$ $SU(N)$ SYM is a conformal field theory (CFT), with gauge coupling g . Its magnetic description, also an $\mathcal{N} = 4$ conformal field theory, has gauge group $SU(N)/\mathbf{Z}_N$ and coupling $1/g$; thus, if $g \gg 1$, the magnetic description is weakly coupled. Adding masses μ to all but the fields of $\mathcal{N} = 1$ SYM causes the scalars of the magnetic description to condense, breaking the dual gauge group completely [14, 15]. This

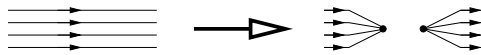


Figure 3. The annihilation of N flux tubes using baryon vertices; this transition would be inhibited if a good dual description of YM theory involved $U(1)^{N-1}$.

non-Abelian Higgs model has string solitons, but because the fundamental group of $SU(N)/\mathbf{Z}_N$ is \mathbf{Z}_N , these strings carry a \mathbf{Z}_N charge [1], in contrast to the integer charges found in the case of broken $\mathcal{N} = 2$ SYM. In short, the electric description of the theory involves confinement by electric flux tubes carrying \mathbf{Z}_N electric flux, leading to a single Regge trajectory, in agreement with expectations for $\mathcal{N} = 1$ SYM. Notice that the associated dual description does not resemble abelian projection: it is essential for the \mathbf{Z}_N charges of the strings that the dual theory is *non-abelian*. Since one cannot obtain the dual $SU(N)/\mathbf{Z}_N$ theory by a projection on the $SU(N)$ theory — the relation between the two is fundamentally quantum mechanical — it seems to me that abelian projection is disfavored.

However, one cannot carry this logic all the way to the $\mathcal{N} = 1$ SYM theory itself. To do so requires taking $\mu \rightarrow \infty, g \rightarrow 0$, but in this limit the magnetic theory becomes strongly coupled and the semiclassical discussion of the previous paragraph becomes unreliable.

What has been obtained here? Two qualitatively different descriptions of confinement have been found, and neither can be continued directly to the theory of interest. One looks similar to abelian projection, while the other absolutely does not. What are we to make of this situation, and how are we to reconcile apparent contradictions?

I believe the correct way to view this situation² is the following. Consider the space of theories in the same universality class as SYM, Fig. 4. Although all of these have a gap, confinement, and chiral symmetry breaking, only theories near a

²I thank N. Seiberg for discussions on this point.

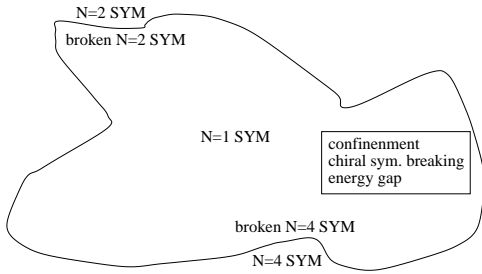


Figure 4. The universality class containing $\mathcal{N} = 1$ SYM, with $\mathcal{N} = 2, \mathcal{N} = 4$ SYM lying just outside.

phase boundary, at the edge of the space, may be expected to have a weakly-coupled Landau-Ginsburg-type description. These dual descriptions may be used to establish the universal properties of $\mathcal{N} = 1$ SYM. Theories far from the boundary, such as $\mathcal{N} = 1$ SYM itself, may simply not have any such description, and so there may not be any weakly-coupled effective theory for describing the string charges, hadron spectrum, and confinement mechanism of SYM. The same logic may apply to non-supersymmetric YM.

In the end, then, our conclusions are very weak — we can show SYM is confining, but we cannot really say much about the mechanism which confines it. Any monopole description of confinement in SYM or YM is likely to be strongly coupled. Is such a description useful? or unambiguous? It seems unlikely to be predictive, in any case. It may be disappointing, but it appears likely there is no simple magnetic description of confinement in nature.

2.4. Breaking Supersymmetry

What happens if we break supersymmetry by adding a mass m to the gaugino?

$$\mathcal{L} \rightarrow \mathcal{L} + m\lambda\lambda \quad (4)$$

The degeneracy of the N vacua will be broken, and the one with lowest energy will depend on the phase of m . As $\theta \rightarrow \theta + 2\pi$, the preferred vacuum will shift from one to the next. All domain walls become unstable (except for $\theta = (2n+1)\pi$, where the two lowest vacua become degenerate.)

However, the energy gap, the spectrum, and the general features of confinement will not be altered if $|m| \ll |\Lambda|$.

A question for lattice study is whether there are any qualitative transitions in the physics of the theory as m increases and pure YM is recovered. For example, consider $T_k(m)$; does it change continuously as a function of m ? Are the confining flux tubes essentially similar in YM and SYM? It would also be interesting to understand the behavior of $\langle \lambda\lambda \rangle$ as a function of m . If no dramatic transitions are seen, then the issues discussed earlier in regard to SYM apply also to YM, in fact to broken $\mathcal{N} = 1$ SYM for all values of m .

2.5. Linkage of YM to $\mathcal{N} = 4$ SYM

Most physicists outside of the field of supersymmetry are inclined to think of supersymmetric theories, especially those with extended supersymmetry, as esoteric curiosities with no real importance for physics. I now intend to convince you that this is far from the truth, and that, in fact, there is a direct link between the spectacular properties of $\mathcal{N} = 4$ SYM and the properties of ordinary non-supersymmetric YM.

Consider the linkage diagram in Fig. 5. We begin at the top with $\mathcal{N} = 4$ $SU(N)$ SYM theory, a conformal field theory with a gauge coupling g . Under Montonen-Olive duality, this theory is rewritten, using magnetic variables, as $\mathcal{N} = 4$ $SU(N)/\mathbf{Z}_N$ SYM, with gauge coupling $\tilde{g} \propto 1/g$. Next, as discussed earlier, we take $g \gg 1$ and break $\mathcal{N} = 4$ supersymmetry to $\mathcal{N} = 1$ by adding finite masses μ for some of the fields. The resulting theory is strongly coupled, confines, and has chiral symmetry breaking, as is easily seen using the weakly-coupled magnetic variables, where the effect of the supersymmetry breaking is to cause the light monopoles to condense, breaking the gauge group completely, and leading to \mathbf{Z}_N -carrying solitonic flux tubes.

The following step is to take g small and μ to infinity, holding the strong coupling scale fixed. The theory becomes pure $\mathcal{N} = 1$ SYM in this limit, and maintains confinement and chiral symmetry breaking since it remains in the same universality class.³ The magnetic variables become

³We know this because the theory is holomorphic in the

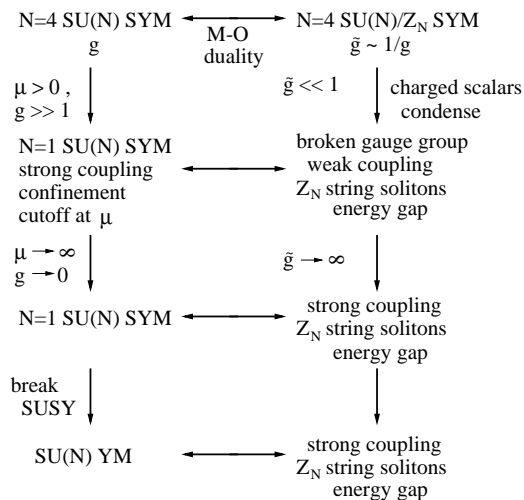


Figure 5. The connection of Montonen-Olive duality to confinement in Yang-Mills theory.

strongly coupled as $g \rightarrow 0$, but we expect the \mathbf{Z}_N solitonic strings will survive, since their existence is a consequence of the topology of the $SU(N)/\mathbf{Z}_N$ gauge group.

The last step is to break $\mathcal{N} = 1$ SYM to YM. Here I must assume that there are no sharp transitions between these two theories — an issue which can be addressed on the lattice, as I discussed above — in order to make the linkage complete. However, because SYM and YM share many properties, such a conjecture is quite plausible. If the transition between these two theories is smooth, then confinement and the \mathbf{Z}_N flux tubes will survive from one theory to the other.

In summary, modulo the conjecture that $\mathcal{N} = 1$ SYM and YM are continuously connected, the specific structure of duality in $\mathcal{N} = 4$ SYM theories is directly related to — perhaps even implies — the fact that YM is a confining theory with \mathbf{Z}_N flux tubes.

parameter μ ; see [5].

3. Gauge Theories with Matter

Supersymmetric gauge theories teach us that the matter content of a theory plays an essential role in determining its basic physics. In particular, the phase at zero temperature of a gauge theory (that is, its properties in the far infrared) depends in a complicated way on (1) its gauge group G , (2) its matter representations R , and (3) the self-interactions \mathcal{L}_m of the matter, including non-renormalizable interactions. Recent work has shown that the phase structure of $\mathcal{N} = 1$ supersymmetric theories is complex and intricate, and it is reasonable to expect that the same will be true of non-supersymmetric theories.

Among the surprises discovered in the supersymmetric context are that there are new and unexpected phases unknown in nature or previously thought to be quite rare. It also appears that duality is a fundamental property of field theory (and also of gravity, and even between gravity and gauge theory!) Certain accepted or at least popular lore has been refuted as well: the beta function does *not*, by itself anyway, determine the phase of a gauge theory; confinement does not always cause chiral symmetry breaking; the 't Hooft anomaly matching argument in favor of such chiral symmetry breaking in strongly coupled theories can be evaded; and fixed points in four dimensions are not at all rare — in fact, they are commonplace!

3.1. SQCD

Supersymmetric QCD consists of $\mathcal{N} = 1$ $SU(N)$ SYM along with N_f flavors of massless scalars (squarks) and fermions (quarks) in the fundamental representation. Fig. 6 shows the phase diagram of the infrared behavior of the theory as a function of N and N_f , as determined by Seiberg in [8].

When $N_f > 3N$, the theory has a positive beta function and flows to weak coupling. Since the theory is free in the infrared in terms of the original variables, this is called the free electric phase.

When $3N > N_f > \frac{3}{2}N$, the theory flows toward strong coupling, but the coupling eventually stops running. The low-energy theory is an interacting conformal fixed point. This is called the “non-

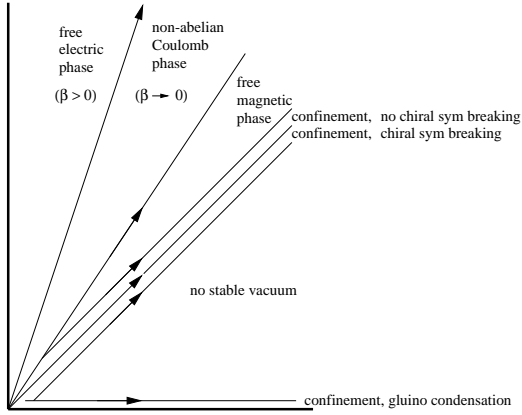


Figure 6. The phases of SQCD.

abelian Coulomb phase”. The theory has a “dual description” using “magnetic” variables.

When $\frac{3}{2}N \geq N_f \geq N + 2$, the theory is in the “free magnetic phase”. The theory flows to strong coupling in the infrared, but the dual variables become weakly coupled. The theory of the dual variables is an infrared-free $SU(N_f - N)$ gauge theory.

For $N_f = N + 1$, the theory undergoes “confinement without chiral symmetry breaking”. The light particles are massless mesons and (scalar) baryons, and the theory describing them is a linear sigma model. For $N_f = N$, the theory displays “confinement with chiral symmetry breaking”. Again the light particles are massless mesons and (scalar) baryons, but now the theory describing them breaks chiral symmetry and becomes a nonlinear sigma model.⁴

When $N_f = N - 1$, instantons generate an unstable potential for the squarks; a similar effect, due to gaugino condensation (or perhaps fractional instantons?) occurs for $N - 2 \geq N_f \geq 1$. For $N_f = 0$, as discussed above, we have confine-

⁴In both these cases the term “confinement” should be taken with a grain of salt, as $SU(N)$ with fundamentals has no phase boundary between confining and Higgs phases, and all flux tubes break due to pair production; however in certain other models confinement is unambiguous, and even there chiral symmetry is not always broken.

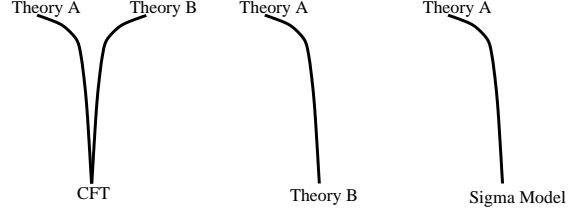


Figure 7. (a) Two gauge theories flow to the same conformal fixed point. (b) Gauge theory A becomes strongly coupled, and at low-energy mysteriously turns into gauge theory B. (c) Gauge theory A becomes strongly coupled, and mysteriously turns into a theory of mesons and baryons.

ment and gaugino condensation.

3.2. Non-Abelian Coulomb Phase

The conformal field theories which are found in this phase are known to exist in perturbation theory at large N and N_f . These perturbative fixed points, often called Banks-Zaks fixed points although they predate those authors, are found both in SQCD and in QCD. We now know that in the case of SQCD the fixed points are found far from large N and N_f . These fixed points exhibit duality: there exist multiple gauge theories which flow to the same conformal field theory, as illustrated schematically in Fig. 7a, and thus multiple sets of variables by which the conformal field theory may be described. This is analogous to Montonen-Olive duality in $\mathcal{N} = 4$ SYM, which is a conformal field theory.

3.3. Free Magnetic Phase

This phase was entirely unexpected. Here, the theory flows to strong coupling, and its infrared physics is described by weakly coupled composite matter and gauge fields. These composites are non-local with respect to the original degrees of freedom, and have an unrelated gauge symmetry. The duality transformation which acts on the fixed points of the non-abelian Coulomb phase operates in the free magnetic phase by exchanging the fundamental theory of the ultraviolet with the infrared-free composite theory, as illustrated

in Fig. 7b. Note that the free magnetic phase is *not* a confining phase, as proven in [20,2].

3.4. Confinement with and without chiral symmetry breaking

In the confining theories, the low-energy description is a theory of gauge singlets built from polynomials in the original degrees of freedom. The main difference between chiral-symmetry-preserving and -breaking theories is in the interaction Lagrangian, which determines the symmetries preserved by the vacuum.

As in the free magnetic phase, the duality transformation exchanges the ultraviolet theory with the infrared one — the quarks and gluons of the gauge theory are exchanged with the mesons and baryons of the linear or non-linear sigma model (Fig. 7c). At this point duality begins to resemble what is done in real-world QCD when we rewrite the theory in term of hadrons and the chiral Lagrangian. This strongly suggests that the QCD/chiral-Lagrangian “duality” transformation is conceptually related to electromagnetic duality, its generalization to Montonen-Olive duality, and its lower-dimensional cousins. I will make this more precise below, using a second linkage diagram; see also [1] for more details.

3.5. Linkage of QCD to Duality in $\mathcal{N} = 2$, $\mathcal{N} = 1$ SQCD

Now I turn to my second linkage diagram, Fig. 8, which relates the duality of finite $\mathcal{N} = 2$ theories to that of $\mathcal{N} = 1$ theories and then to the properties of real QCD.

At the top of the diagram, we have a finite — and therefore conformal — $\mathcal{N} = 2$ SQCD theory, with gauge group $SU(2)$, gauge coupling g , and eight quarks and squarks in the doublet representation.⁵ As in the case of $\mathcal{N} = 4$ SYM discussed earlier, this theory has a representation in terms of magnetic variables as another $\mathcal{N} = 2$ SQCD theory, which in this case has the same gauge and matter content as the electric variables but has coupling constant $1/g$. This duality transformation was discovered by Seiberg and Witten in their famous paper of 1994 [21].

⁵The choice of $SU(2)$ is for simplicity only; the same physics applies with slight modification for any $SU(N)$.

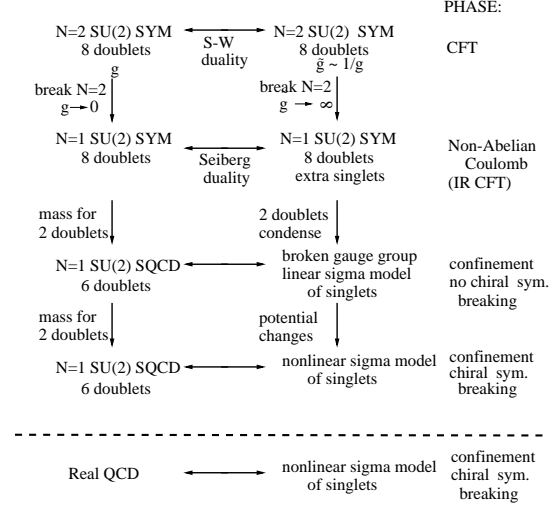


Figure 8. The connection of Seiberg-Witten and Seiberg duality to confinement, chiral symmetry breaking, and non-linear sigma models in QCD.

Now let us break $\mathcal{N} = 2$ supersymmetry to $\mathcal{N} = 1$ by giving mass to the extra fields in the $\mathcal{N} = 2$ gauge multiplet. The theory becomes $\mathcal{N} = 1$ SQCD with gauge group $SU(2)$ and eight quarks and squarks in the doublet representation. This theory has a running coupling, but flows to a conformal fixed point in the infrared — it is in the non-abelian coulomb phase. As shown in [22], the breaking of $\mathcal{N} = 2$ supersymmetry causes the magnetic theory to flow to an $\mathcal{N} = 1$ SQCD theory with the same charged matter content but with extra gauge singlets and interactions, precisely those required by Seiberg’s $\mathcal{N} = 1$ duality transformation [8]. In other words, the Seiberg-Witten duality transformation of the $\mathcal{N} = 2$ theory flows to the Seiberg duality transformation of $\mathcal{N} = 1$ SQCD.

Now add masses for two of the electric doublets, leaving a theory of $SU(2)$ with six doublets. This causes some magnetic squarks to condense, breaking the $SU(2)$ magnetic gauge symmetry and leaving a theory of massless gauge singlet fields M . These singlets are precisely the mesons of the electric variables. Since the mag-

netic gauge symmetry is broken, electric charge is confined. Thus, confinement proceeds through a non-abelian generalization of the dual Meissner effect, and the low-energy magnetic theory — an infrared-free non-renormalizable theory with a cutoff — is the sigma model describing the confined hadrons. Examination of the sigma model, in particular the potential energy $V(M)$, shows that the theory has a vacuum in which chiral symmetry is unbroken.

Adding masses for two more doublets merely causes the potential $V(M)$ to change in such a way that there is no longer a chiral-symmetry-preserving vacuum. Thus, $SU(2)$ SQCD with four doublets confines and breaks chiral symmetry. Shifting to the true vacuum and renaming the fields as representing fluctuations around that vacuum, we may rewrite the theory as a non-linear sigma model of pions.

To go from here to real non-supersymmetric QCD is a bit more of a stretch than in the previous linkage diagram, because the removal of the scalar squarks from the theory is rather more delicate and much less well understood. Rather than raise those questions, I leave the last step in the diagram as more of a heuristic one. It is evident that the duality in $\mathcal{N} = 1$ $SU(2)$ SQCD with four doublets, relating the gauge theory of gluons, quarks and their superpartners to a non-linear sigma model of pions, is remarkably similar to the transformation between real-world QCD and the chiral Lagrangian that we use to describe its infrared physics. Indeed it is completely justified to call this QCD-to-hadron transformation “duality”. As we have seen, the duality in confining SQCD can be derived from the Seiberg-Witten duality of a conformal $\mathcal{N} = 2$ SQCD theory. Is QCD-pion duality likewise embedded in a chain of non-supersymmetric duality transformations similar to those in the diagram?

3.6. Non-Supersymmetric Vectorlike Theories and the Lattice

We are unable at this time to answer this question, even for non-chiral theories like QCD, because almost nothing is known about non-supersymmetric gauge theories other than $SU(2)$ and $SU(3)$ YM and QCD with a small number

of flavors. This is where lattice QCD comes in, as it is at the present time almost the only tool available for studying this issue.

In supersymmetric theories, we have begun to understand the complicated subject of the long distance physics of gauge theories as a function of their gauge group G , their matter representations R_i , and their interaction Lagrangian (including non-renormalizable terms.) Similar information would be welcome in the non-supersymmetric case. We know that the non-abelian coulomb phase exists at large N_c, N_f when the one-loop beta function is very small, but how far does it extend away from this regime? What properties does the theory exhibit as it makes the transition from the perturbative regime to the conformal regime? The confining phase in supersymmetric theories is the exception, not the rule; which non-supersymmetric theories actually confine? Which ones break chiral symmetry, and in what patterns? What are their confining string tensions T_k and their low-lying hadron spectra? Effects involving instantons, fractional instantons, monopoles, *etc.* may play an important role in some theories — but which ones, and what effects are they responsible for? The free magnetic phase may not exist in non-supersymmetric theories — perhaps it requires the massless scalars of SQCD — but it would be very exciting if it were found (and a tremendous coup for the group which demonstrates its existence!) The existence of this phase would be a sufficient but not necessary condition for gauge theory–gauge theory duality in the non-supersymmetric context, which could also perhaps be shown in the context of the non-abelian coulomb phase. And of course, we must not leave out the possibility of new exotic effects which do not occur in the supersymmetric case.

It is important to emphasize that these questions are by no means of purely academic interest. The problem of electroweak symmetry breaking has not been solved, and there remains the possibility that the symmetry breaking occurs through a technicolor-like scenario, in which it is driven by chiral symmetry breaking in a strongly-coupled gauge theory. Technicolor models with physics similar to QCD have been ruled out by precision

measurements at LEP. However, if their physics is considerably different from QCD, then we have no predictive tools, and therefore no experimental constraints. It will be an embarrassment to theorists if the LHC discovers evidence of strong dynamics of a type that we simply do not recognize. It is therefore important for model building and for comparison with experiment that we improve our understanding of strongly-coupled non-supersymmetric theories.

I should mention also that there are potential spinoffs from such a program in the areas of supersymmetry breaking (which can be detected but not quantitatively studied using presently available analytic techniques) and in condensed matter systems where many similar physics issues arise.

To answer these questions, which require studying theories with very light fermions and often a hierarchy of physical scales, will require powerful lattice techniques not yet fully developed. I encourage you all to think about how best to pursue this program of study.⁶

4. Large N_c Gauge Theory and String Theory

The fact that gauge theory, in the limit of a large number of colors, is in some way connected with string theory, with $1/N$ playing the role of the string coupling, was first noted twenty-five years ago by 't Hooft. While many have attempted to make progress in understanding gauge theories by studying this limit further, a quantitative approach to $SU(N)$ YM or QCD using $1/N$ as an expansion parameter has been stymied by the difficulty of determining the classical string theory which should appear in the $N \rightarrow \infty$ limit.

Let us consider some obvious facts about this

⁶A useful testing ground is to be found in three-dimensional abelian gauge theories, both with and without supersymmetry. The supersymmetric theories are known to have an intricate phase structure, with a duality transformation (called “mirror symmetry”) and interesting large N_f behavior. There are examples of non-trivial fixed points, infrared-free mirror gauge theories, chiral symmetry breaking and confinement. Many of these phases are likely to show up in the non-supersymmetric case, and lattice approaches to studying non-supersymmetric QCD could be tested in these theories.

string theory, and the gauge theories which might be studied using this approach. First, critical (that is, $d = 10$) superstrings have gravity and supersymmetry. Since QCD has neither light spin-two hadrons nor supersymmetry, clearly its string theory is unrelated to critical superstrings. A second obvious fact is that only confining theories have physical string-like flux tubes and associated area laws for Wilson loops. Other theories, including conformal field theories such as $\mathcal{N} = 4$ SYM, have no physical string-like behavior and should not be associated with string theories.

We will not get anywhere using these obvious facts, however, because they are wrong. Following on the work of Gubser and Klebanov [9], and followed in turn by work of those authors with Polyakov [11], of Witten [12], and then of a flood of others, Maldacena proposed a bold conjecture [10] relating supersymmetric conformal field theories in four dimensions to superstring theory. I will now state this conjecture, defining my terms as I go.

4.1. Maldacena’s Conjecture

According to Maldacena’s idea, $\mathcal{N} = 4$ SYM theory with N colors and coupling g is related to Type IIB superstring theory (a ten-dimensional theory of closed strings with IIB supergravity as its low-energy limits — its massless fields are a graviton $G_{\mu\nu}$, antisymmetric tensor $B_{\mu\nu}$ and dilaton ϕ along with “Ramond-Ramond” 0-index, 2-index and 4-index antisymmetric tensor fields $\chi, A_{\mu\nu}, C_{\mu\nu\rho\sigma}$.) The string theory exists on a ten-dimensional space consisting of a five-sphere ($x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 = R^2$, a space of constant positive curvature) times a five-dimensional Anti de Sitter space ($-x_1^2 - x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 = R^2$, a space of constant negative curvature) with non-zero flux of $\partial_\kappa C_{\mu\nu\rho\sigma}$. The radius of the sphere and of the AdS space are both $R = g^2 N$, so the curvature of the space is small at large $g^2 N$, while the string coupling g_s is the square of the gauge coupling $g^2 = R/N$. (Notice the N dependence accords with 't Hooft’s original observation.)

Where is the four-dimensional gauge theory in this ten-dimensional string? The answer is fascinating. The five-dimensional AdS space has a

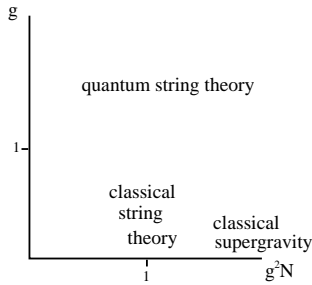


Figure 9. The conjectured duality between superstring theory and gauge theory connects gauge theory to classical superstring theory at small g and further to classical supergravity at large g^2N .

boundary, an infinite spacelike distance away but at finite lightlike distance, and it is there that the gauge theory is to be found. Note that it has long been understood from the work of Polyakov that non-critical strings dynamically grow an extra dimension, so the presence of a five-(plus-five)-dimensional string theory in the context of a four-dimensional gauge theory is perhaps not so shocking. What is astonishing is that the string theory involved is the well-understood critical superstring, and that it is believed to be *equivalent* to the gauge theory on the boundary.

This conjecture is not yet known to be true, but it has been tested in the large N , large g^2N regime, in which the full quantum string theory reduces to classical (small g_s) supergravity (small curvature), as shown in Fig. 9. In this limit, the symmetries and operators of the two theories match [10]; and the baryon operator (which is non-perturbative in string theory since $N \sim 1/g_s$) has been identified with a D-brane (a soliton) [23]. Furthermore, the Wilson loop of the gauge theory has been identified with the boundary of a string worldsheet in the larger space [24,25]. This makes it possible to explain how this conformal $\mathcal{N} = 4$ SYM theory can be stringy: although the Wilson loop is the boundary of a string, the string does not live on the boundary but hangs into the bulk, and so the value of the Wilson loop as a function of its size *depends on the geometry out-*

side the boundary. The difference between area law and perimeter law in various gauge theories thus is translated into differing spatial geometries in the string theory duals of those gauge theories.

These ideas have been further extended in a number of directions. In particular, it is easy to study finite temperature, and to use the fact that high-temperature five-dimensional SYM has infrared behavior equivalent to four-dimensional non-supersymmetric YM at strong coupling [26–28]. It is straightforward to show that this strongly coupled theory confines. It is also possible to compute its spectrum of glueballs [29]. Remarkably, the ratios of certain glueball masses match rather well to lattice results in ordinary QCD away from strong coupling, both in three and four dimensions. However, although this is surprising and possibly an interesting statement about this particular strong-coupling limit, there is no clear reason for great excitement. There are many extra states in the spectrum which do not arise in YM theory; the glueball mass is not naturally related to the string tension; and there is no systematic approach toward recovering real YM theory starting from this limit.

Other extensions include adding matter (which leads to both open and closed strings), reducing supersymmetry, changing the number of dimensions, and studying non-conformal theories at zero and finite temperature. Various expected phase transitions, such as deconfinement at high temperature, have been observed.

However, serious obstacles lie in the path of any attempt to apply these string theory techniques to YM or real QCD, where for any fixed N the value of g runs such that g^2N is not always large and g is not always small. Where g^2N is not large, supergravity is insufficient and string theory is required for the large N limit. Unfortunately, almost nothing is known about string theory on an AdS background with Ramond-Ramond fields, even at the classical level, since the usual worldsheet formulation of string theory cannot be easily generalized to this case. Even were this problem solved, there is no guarantee that the solution will be easy to use. I cannot tell you whether these obstacles will be overcome tomorrow, next year, or in the fourth millenium; but in any case

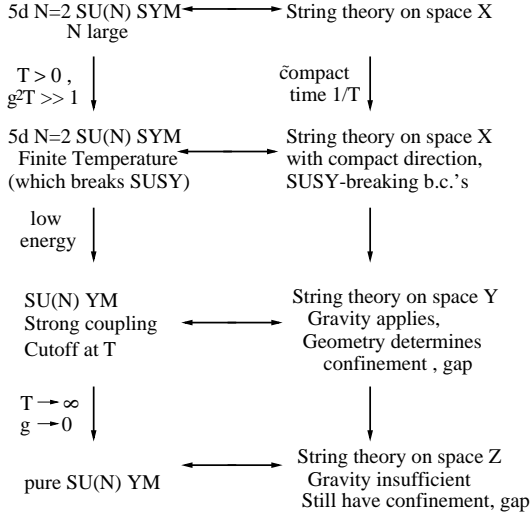


Figure 10. The connection of the Maldacena conjecture generalized to five dimensional $\mathcal{N} = 2$ super-Yang-Mills theory to confinement in Yang-Mills theory.

the difficulties are such that it seems unlikely these approaches will become a quantitative competitor to lattice gauge theory in the near future.

Nonetheless, it is remarkable that a sensible and definite proposal for the large N expansion of gauge theory has been made and has passed some non-trivial tests — and that it appears to involve superstring theory! And we must certainly ask whether in fact superstring theory can be *defined* using gauge theory.

4.2. A Final Linkage Diagram

To again bring home how apparently esoteric results on theories with extended supersymmetry can have implications for real-world physics, I want to restate using Fig. 10 the connection between QCD and string theory as presently conjectured and partially understood. I must warn the reader that I will be speak rather loosely when describing this diagram, as this discussion is intended for novices who want only a rough idea of the physics. I ask experts to forgive the obvious misstatements.

Let us consider Euclidean five-dimensional $\mathcal{N} = 2$ SYM theory (under compactification to four dimensions this would become $\mathcal{N} = 4$ SYM.) According to the Maldacena conjecture this theory is dual to a superstring theory which in the large N limit approximately becomes supergravity on a certain space. Compactifying the time direction using periodic boundary conditions for bosons, antiperiodic boundary conditions for fermions, gives us on the one hand a finite-temperature version of the SYM theory, and on the other the same supergravity theory on a space with a compact time direction and supersymmetry-breaking boundary conditions. At energies below the temperature only the bosonic modes invariant under the compact time remain; the gauge theory appears as four-dimensional and non-supersymmetric YM, while the gravity is again just the same theory with the Kaluza-Klein modes removed. The temperature T serves as a cutoff on the YM theory (since there are many Kaluza-Klein modes with masses of order T) and the coupling $g(T)$ of the YM theory must be very large at the energy scale T if supergravity is to be a good approximation. If we wish to take T to infinity and the YM coupling $g(T)$ to zero, holding Λ_{YM} finite, then the supergravity regime breaks down and we must include the full details of the string theory — a superstring theory with explicit supersymmetry breaking effects from the finite temperature construction.

The first step of this linkage is understood. The last, however, is a chasm badly in need of a bridge.

5. Summary

I have shown you today that many of the developments in the seemingly abstract field of supersymmetric field theory have direct or indirect implications for non-supersymmetric YM and QCD. $\mathcal{N} = 1$ SYM is a theory with properties likely to be shared with non-supersymmetric theories. Predictions for this theory can be tested on the lattice, and the transition between SYM and YM is of considerable theoretical interest. $\mathcal{N} = 1$ SQCD, as a function of its gauge group, matter content, and interactions, shows a wide variety of phenomena, many of which might appear in non-

supersymmetric QCD. Only on the lattice can we search for these phenomena, some of which might actually play a role in nature. And the connection between the large N limit of YM and QCD and string theory, known for many years, now appears to be the tip of a large iceberg relating gauge theories and gravity/string theories. Although quite preliminary, this connection has the greatest potential — still far from realized — to impact our understanding of physical QCD.

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